JOURNAL OF EDUCATION, SOCIETY & MULTICULTURALISM
Valahia University of Targoviste, ROMÂNIA
Department of Education Sciences

Formation of mathematical representations in primary school students

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Abstract

In the process of representations' forming, schematizations, nuances, simplifications are involved along with the perceptual syntheses and also a series of regroupings of the features of the person or the object evoked. The contribution of the study is highlighted in the theoretical basis of the term ***representation*** *in the field of psychology and mathematics teaching. The purpose of this paper was to investigate teachers' opinions on the process of mathematical representations' forming. Respondents are primary school teachers who have voluntarily agreed to take part in the survey. The research tool is a questionnaire created through Google Forms and applied online. The obtained results highlight the positive vision of the respondents on the interdisciplinary approach in the formation of mathematical representations.*

Keywords: *representation, mathematics, teachers;*

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1. Introduction

The mental universe of people is populated with representations. We experience *representation* as a flow of internal images and gestures. The words and symbols we use to communicate do not refer directly to reality, but to represented entities: objects, relationships, processes and actions between which there is no automatic agreement.

An important concept of cognitive psychology, *representation* spreads to all sectors of knowledge analysis and especially on the teaching of mathematics. We wonder if this propagation has drawbacks, as the meanings of this term in mathematics and psychology are different.

2. Theoretical background

In cognitive psychology, the term “representation” is accompanied by an adjective, appearing as “mental representation” (Rupert, 2018, p. 205; Schack & Frank 2020, pp. 1-2). Mental representation is a "cognitive entity that reflects in the mental system of an individual a fraction of the outer universe of this system" (Denis, 1999, p.779).

In Romania, the literature dedicated to defining the concept of representation is vast and complex. Representation is a process that "allows mental action with the object in its absence, provided that it had previously acted on our sense organs" (Zlate, 1994, p.105), "the ability to re-construct an image in the absence direct contact with the object ”(Neculau et al., 2005, p. 31),“ the first step on the way of

detaching the act of knowledge from the immediate material and its orientation towards abstract and general "(Lupșa & Bratu, 2005, p. 20), or as a "sensory cognitive process of signaling in the form of a unitary but schematic image of the material and characteristic features of objects and phenomena in the absence of their direct action on analyzers" (Sterian, 2017, p. 43).

Representation is defined not only as a process, but also as a "secondary product (image), as opposed to the primacy" (Popescu-Neveanu, 1978, p. 617), a constructive process, as it facilitates the mental appearance of unitary images, images that can be modified according to the requirements imposed by mental and practical activities (Zlate, 2005, p. 36).

Currently, in cognitive psychology, a meaning with a similar meaning is used - "mental image" defined as "a reflection in an internal environment of external reality" (Miclea, 2003, p.26). Mental images dynamically reflect aspects related to the shape of concrete objects, their position in space or the development of a certain phenomenon of nature (Sterian, 2017, p. 44). The mental image can be reflected as an imitation, acquired in the presence of a model, the difference being made between the reproductive image and the anticipatory image. Thus, the image can acquire a capacity for anticipation, for the internal reproduction of a movement that is to appear, proving relevant for human existence (Zlate, 1994, pp. 113-114), but also a real support in building the meaning of a word. (Zlate, 2005, p.39).

In the didactics of mathematics, the importance of clarifying the notion of representation is obvious (Amato, 2008, p.177; Abbey, 2016, pp.16-18). Empirical studies conducted in this field emphasize the importance of direct operation with teaching materials (Golafshani, 2013, p. 137; Souillard, 2016, pp. 16-17), of virtual tools (Pierce & Stacey, 2013, p. 323; Bouck, Flanagan , Bouck, 2015, p. 382; Tricot & Chesné, 2020, p.19) or with materials made by students, Another study is focused on the semiotic analysis of mathematical activities (Houdement & Petitfour, 2018, p. 9). Also, the presence of some confusions between the terms used in the common vocabulary and the mathematical field of the concepts of figure and number is mentioned (Goudenhooft, 2018, p. 4).

The term *representation* refers to both the process and the product, the act preceding the understanding of mathematical concepts. The term is used formally, but always accompanied by the specification: "graphic representation", "geometric representation" (Red, 2007, pp. 10-12) and most often informally, for example in expressions such as "*a* is a representative of the A set".

Although the need to develop mathematical thinking is strongly emphasized in didactics, the transition from intention to achievement proves to be difficult. In the case of many children, the failure of learning is generated by a conception that is still dominat today in Maths primary classes: mathematical thinking is the spontaneous result of training, an automatic consequence of information storing. How mathematical content is represented is fundamental to how students understand and use that content. The

formation of mathematical representations is not limited to the registration of information and its name. These are provided in a systematic way, constituting as many opportunities for activating the process of thinking (Muraru, 2020).

In the context of the changes brought by the application of the new National Curriculum for primary education (MEC, 2019, p. 3), we consider it appropriate for the teacher to design innovative, interdisciplinary learning activities that facilitate the formation of representations among school children, in precise circumstances, for and through students.

3. Methodology

3.1. Objective

The present research study aims to investigate the opinions of primary school teachers on the formation of mathematical representations of students.

3.2. Participants

The participants in this study were 397 primary school teachers, employed in the public system. 96.50% of respondents are female, and 3.50% of respondents are male. The analysis of the variable related to the school's residential environment shows that 76% of the respondents work in urban areas. The analysis of the variable regarding the didactic degree held by the respondents shows that 2% are beginner teachers, 13.85% have the finalized the first activity year, 34% are teachers finalizying second didactic degree, and 50.10% are

teachers with the first didactic degree . From the analysis of the data regarding the seniority of respondents in education, it turns out that 1% of the respondents have less than 1 year teaching experience, 9.57% have between 1 and 5 years teaching practice, 10.07%, between 6 and 10 years, 20.65% - between 11 and 15 years, 27.70% - between 16 -20 years, and 30.98% of respondents have over 20 years of teaching practice. The data analysis shows that most respondents coordinate students in class I, the sample being described as follows: 10.3% of respondents coordinate students enrolled in preparatory class, 50.9% in the first grade I, 23.7% in the second grade, 5% in the third grade, and 10.10% in the fourth grade.

3.3. Instrument

The research tool was a questionnaire with 12 questions regarding the forms of improvement in mathematics teaching, the concept of *representation*, the formation of mathematical representations, opinions on the process of formation of representations in primary education and 5 socio-demographic questions.

3.4. Procedure

The research was conducted in the first semester of the 2020-2021 school year. The questionnaire was developed on the basis of an extensive literature review and was adapted for the purpose of the research. The questionnaires were completed online. The participation of

the respondents was voluntary. It took about 20 minutes to complete the questionnaire. The anonymity of the participants was maintained.

4. Results and Discussion

The existing correlations between the opinions regarding the current situation (current school year) regarding the improvement in the field of formation of mathematical representations in students and the ideal education environment were analyzed (table 1).

Table 1. Analysis of correlations related to the forms of improvement in the didactics of mathematics

The current situation (current school year) regarding the improvement in the field of formation of mathematical representations at students	**The situation you prospect in the future in terms of training in the field of formation of mathematical representation at students**
Q.1.1.1. the possibility to take part in continuous training courses in mathematics teaching this school year;	Q.1.2.1. the possibility to take part in continuous training courses in mathematics teaching this school year;
Q.1.1.2. the possibility to design methodological-scientific activities carried out within the methodical commissions this school year;	Q.1.2.2. the possibility to design methodological-scientific activities carried out within the methodical commissions this school year;
Q.1.1.3. the possibility to present papers and works at scientific communication sessions;	Q.1.2.3. the possibility to present papers and works at scientific communication sessions;
Q.1.1.4. the possibility to consult specialists in the field of mathematics teaching;	Q.1.2.4. the possibility to consult specialists in the field of mathematics teaching;

The determination quotient between Q.1.1.1. and Q.1.2.1. is 52.27% of Q.1.1.2. and Q.1.2.2. is 50.98%, between Q.1.1.3.and Q.1.2.3. is 49.84%, and the one between Q.1.1.4. and Q.1.2.4. is 58.06%. The analysis of the results obtained for the correlation between the four forms of improvement in mathematics teaching confirms the respondents' interest in informing themselves and in contributing to the increase of the qualitative level of the educational services provided.

The statement regarding the selection of a single definition of the concept of representation (Q.2.1.) received the following percentages: “Q.2.1.1. the psychic process that allows the mental action with the object in its absence ”- 23%,“ Q.2.1.2. a constructive process that facilitates the mental appearance of images that can be modified according to the requirements imposed by daily activities "- 32%," Q.2.1.3. steps towards the development of a structured thinking ”- 19%, and“ Q.2.1.4. expressing a relationship between two structures ”- 31%. These results allow us to state that the respondents relate in defining the concept of representation both in psychology and in the teaching of mathematics.

Statement coded Q.2.2. aims at selecting a characteristic of representations, considered by the respondents as the most important in the learning process. "Q.2.2.1. simbolic meaning ”recorded 61.2% answers,“ Q.2.2.2. promptness ”- 35.3%,“ Q.2.2.3. panning ”2.3% and only 1.3% -“ Q.2.2.4. autonomy ". "Q.2.2.5. social character ”received no response.

The answers given by the teachers regarding the ordering of the modalities through which the teachers support the students to form their best representations (Q.2.3.) indicate the following results: „Q.2.3.1. learning starts from students' previous knowledge ”- 47.4%,“ Q.2.3.2. integrated approach of the concepts specific to the fields of Mathematics and Natural Sciences ”- 31, 2%,“ Q.2.3.3. presentation of the topic in close connection with the experiences of daily life ”- 18.92%,“ Q.2.3.4. creating conditions for self-discovery and learning ”- 2.3%, and“ Q.2.3.5. guiding students to the creative application of what they know ”- 0.3%.

The answers offered by the subjects of the research sample to the assurance in the learning process of some concrete objects, meant to support the formation of the best possible representations exceed the value 3.8, in other words over 90% of the respondents chose the answers "quite often" and "always." Thus, for each item, the statistical averages are the following: “Q.3.1.1. analytical / synthetic determination of the set ”- 3.88,“ Q.3.1.2. composing and decomposing natural numbers ”- 4.07,“ Q.3.1.3. performing operations with natural numbers ”- 3.92,“ Q.3.1.4. construction of simple regularities ”- 4.05, and for“ Q.3.1.5. problem solving ”- 4.07.

The answers to the question “Q.3.2. What would be the most appropriate type of training for the formation of mathematical representations in primary school students? ” shows that the most frequent training is the interdisciplinary type 90.4%, a percentage justified otherwise by the curriculum of the discipline.

Among the completed answers to the statement “Q.3.3. Indicate an own action that you consider appropriate in the formation of mathematical representations in primary school students ”we list the following codes: use of concrete objects - 121 answers, use of materials in electronic format - 94 answers, use of worksheets - 76 answers, activities through didactic didactic games - 106 answers.

Statement Q.2.5 allows the analysis of the extent to which the mentioned descriptors ensure the formation of mathematical representations in students. Thus, a high frequency is given by the descriptors “Q.3.4.1. interest in exploring mathematical regularities and relationships encountered in familiar situations ”and“ Q.3.4.3. problem solving through trial and error strategies; analysis and reflection on some life situations, problems ”- 84.2%. Another relevant aspect for the formation of quality representations is given by the support of experienced teachers. When they encounter difficulties in the process of representationsț forming for primary school students, the respondents thus turn to school colleagues-47%, the head of the methodical commission "-43%, the specialized inspector" -9% and a very small percentage to specialists in didactics - 1%.

The open answers completed by the teachers to the item regarding the improvement of the process of training the representations to the students from the primary education show, once again, the importance given to the training courses carried out through the Houses of the Didactic Board -78%. An effective way to disseminate examples of good practice in the formation of representations was the development of open educational resources, in

electronic format, posted on the official pages of educational institutions -81%.

5. Conclusion

In conclusion, multiple representations and multimedia support learning. In the formation of quality representations in students, teachers have an important role.

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